IV Kongres Statystyki Polskiej

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Bayes Risk Consistency of Nonparametric Classification Rules

for

Point Processes

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Outline

I Spike Trains Data(Spiking NN) and Point Processes

II Classification for Point Processes

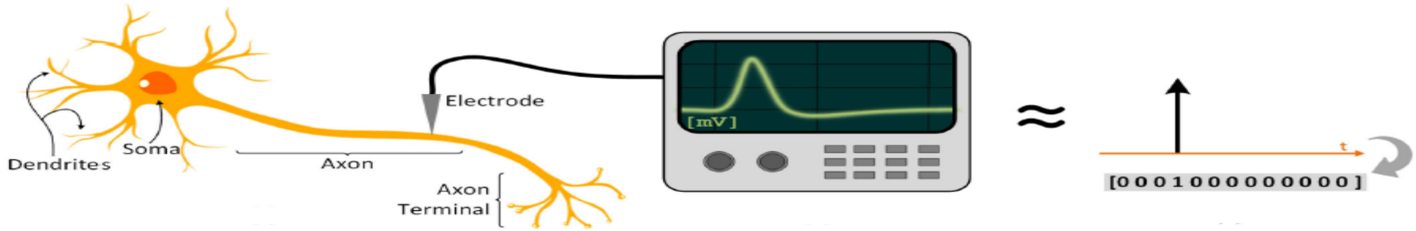
1. **Bayes Rule and Risk**
2. **Nonparametric Plug-In Classification Rules**
3. **Kernel Classifiers**

III Extensions

I Spike Trains Data(Spiking NN) and Point Processes

Spike Trains Data

- **Biological Systems**



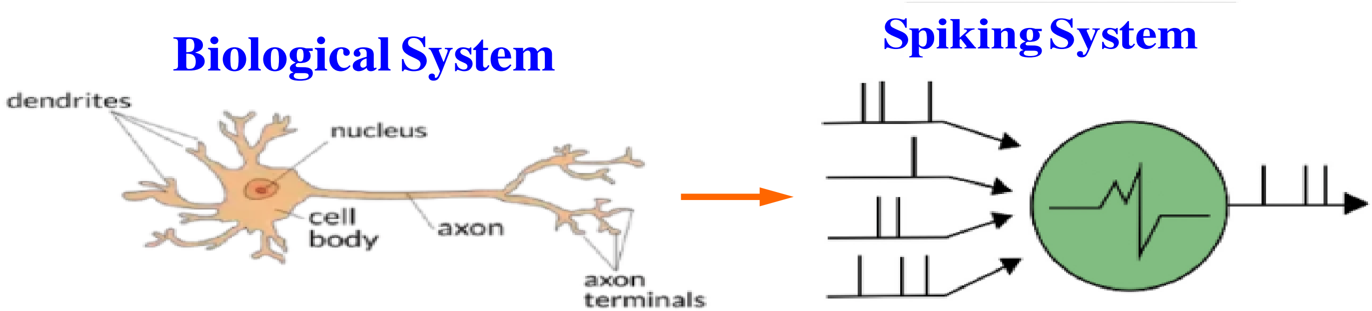
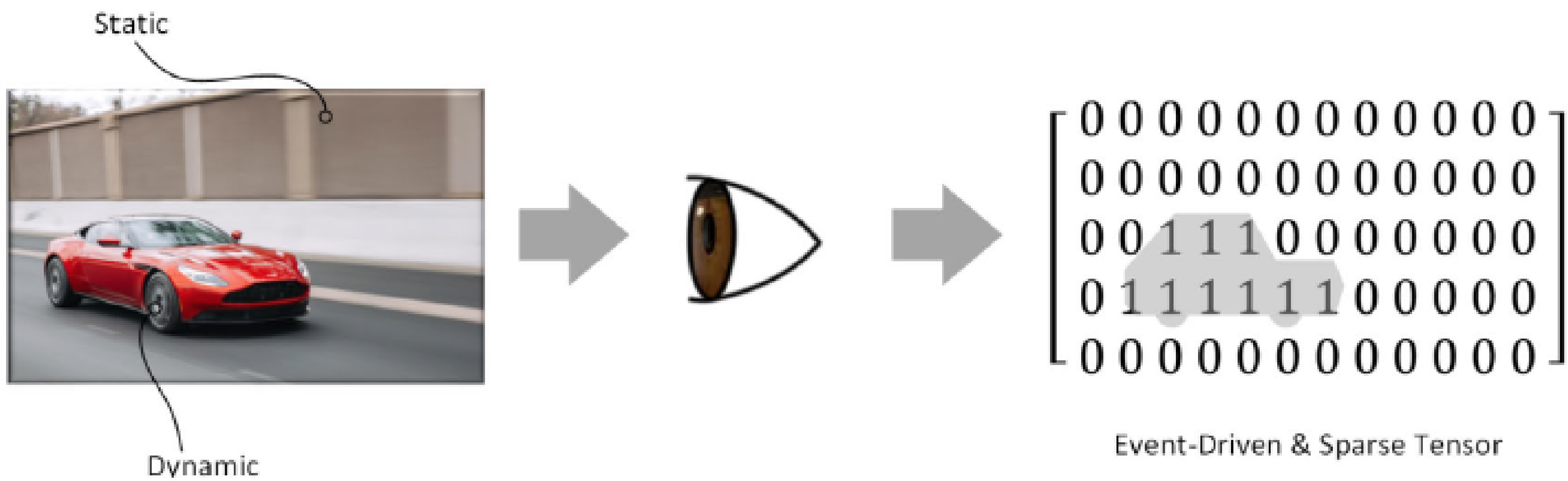
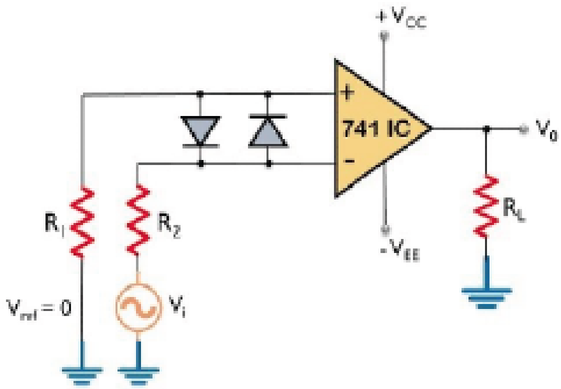


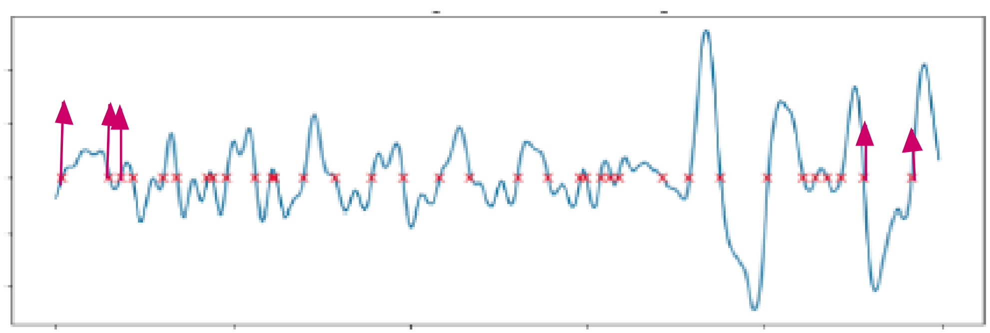
Image Analysis

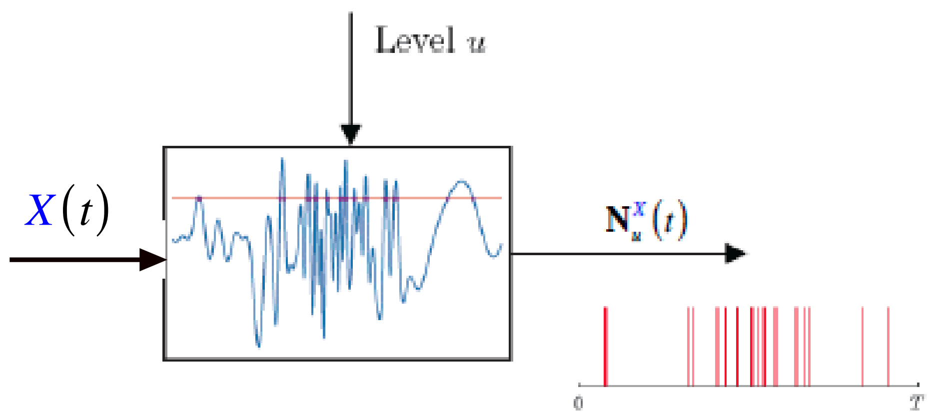


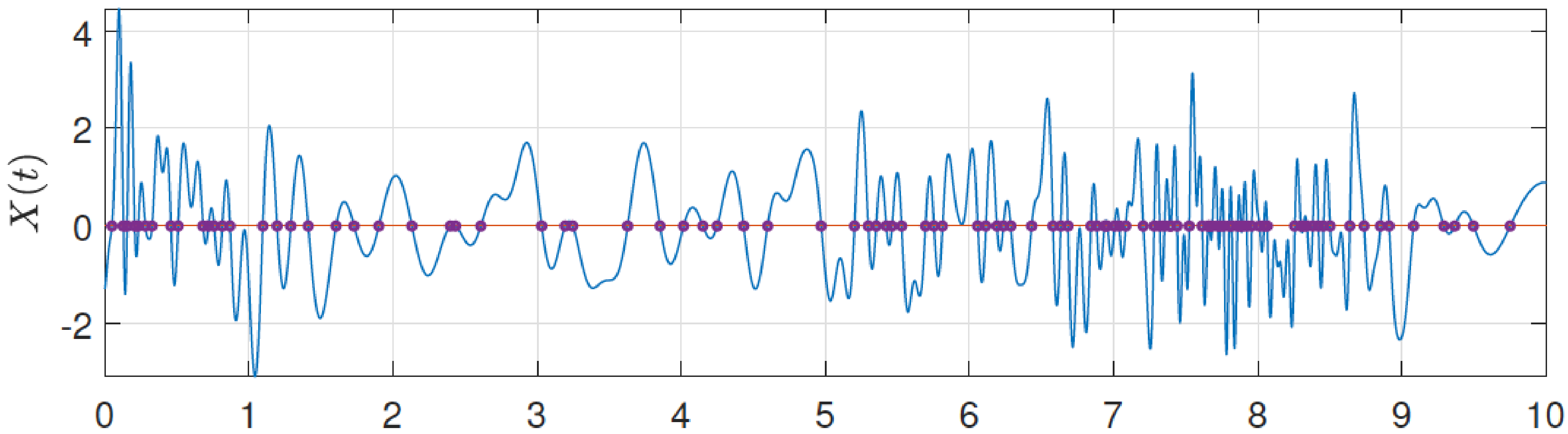
Event Driven Signal Analysis: Level Crossings



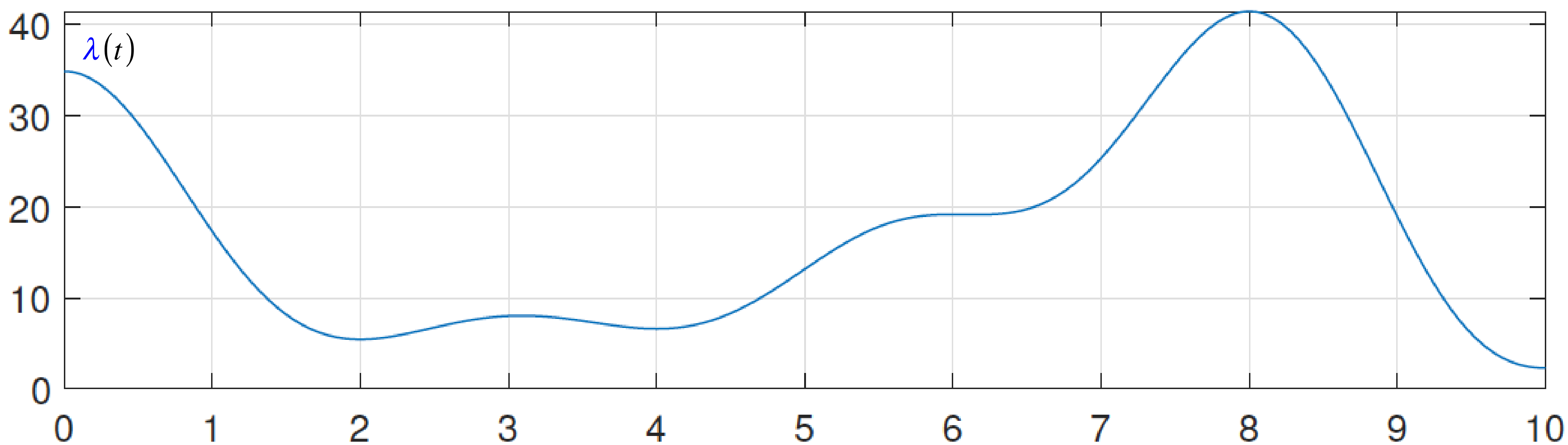
Circuit Diagram of Zero-Crossing Detector



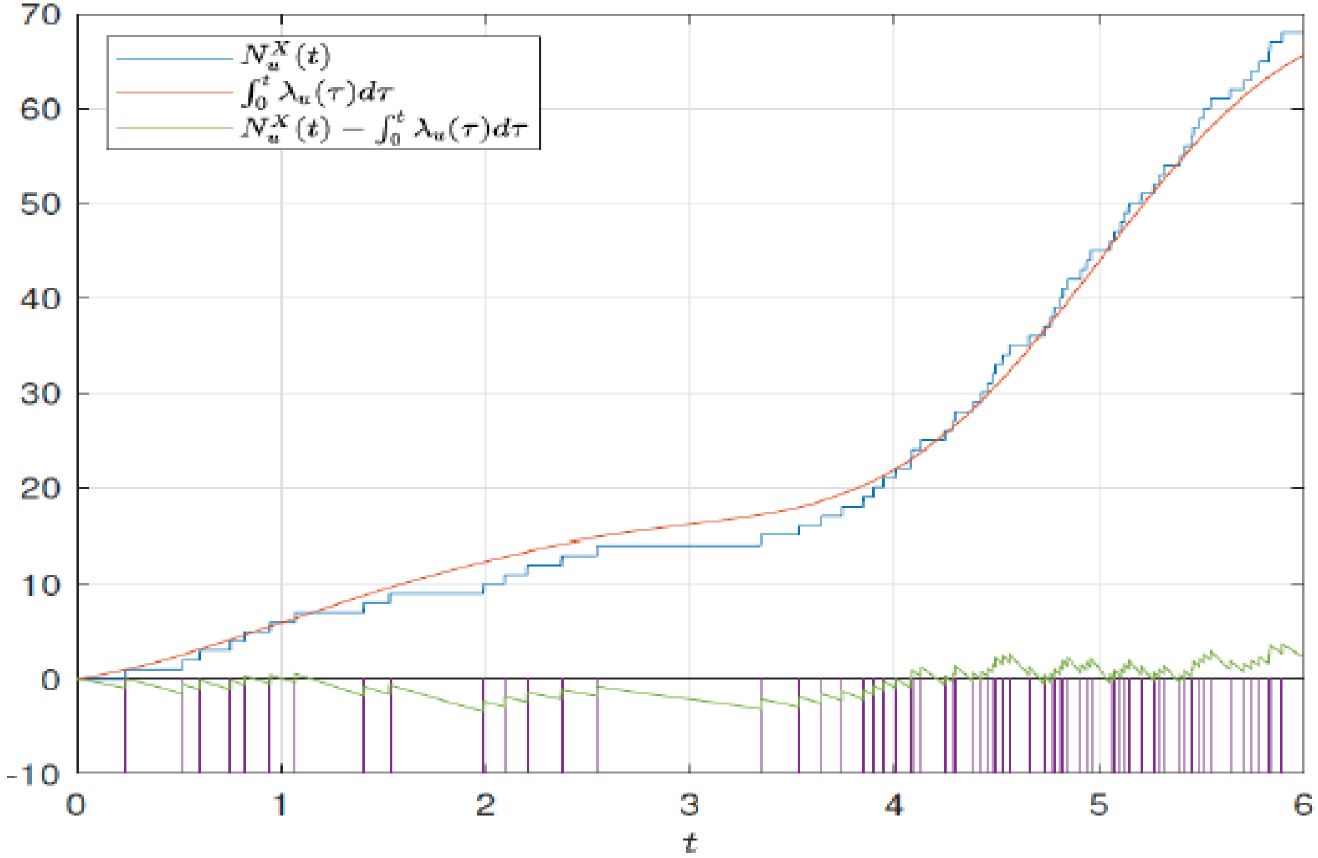




Local Intensity



▲ **Level Crossings Counting Process**



- **Inference from Level Crossings**

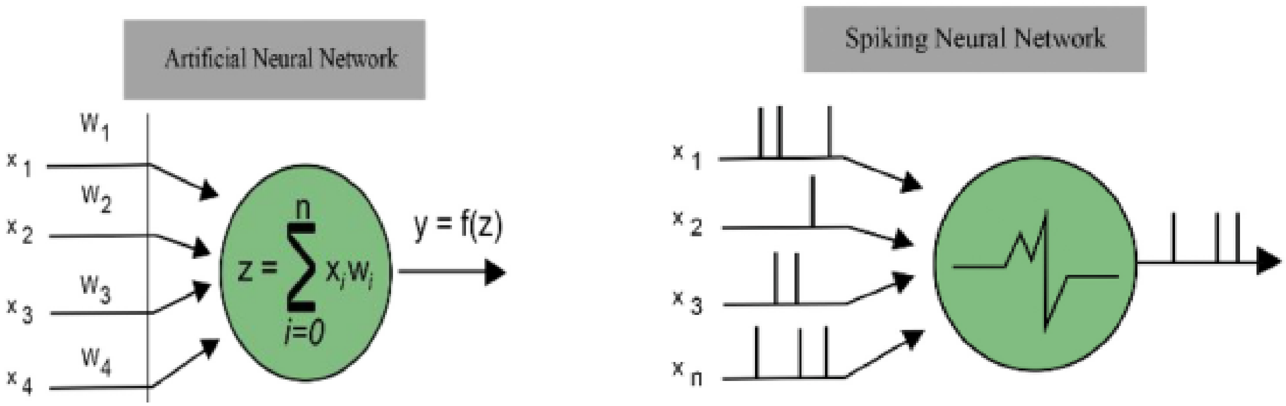
  Lindgren: Statistical Science, 2019

  Pawlak: IEEE Signal Processing, 2023

Spiking Neural Networks

Networks of Spiking Neurons: The Third Generation of Neural Network Models

Wolfgang Maass



2022 roadmap on neuromorphic computing and engineering

Dennis V Christensen[1], Regina Dittmann[2], Bernabe Linares-Barranco[3], Abu Sebastian[4], Manuel Le Gallo[4], Andrea Redaelli[5], Stefan Slesazeck[6], Thomas Mikolajick[6,7], Sabina Spiga[8], Stephan Menzel[9], Ilia Valov[9], Gianluca Milano[10], Carlo Ricciardi[11], Shi-Jun Liang[12], Feng Miao[12], Mario Lanza[13], Tyler J Quill[14], Scott T Keene[15], Alberto Salleo[14], Julie Grollier[16], Danijela Marković[16], Alice Mizrahi[16], Peng Yao[17], J Joshua Yang[17], Giacomo Indiveri[18], John Paul Strachan[19], Suman Datta[20], Elisa Vianello[21], Alexandre Valentian[22], Johannes Feldmann[1], Xuan Li[23], Wolfram H P Pernice[24,25], Harish Bhaskaran[23], Steve Furber[26], Emre Neftci[27], Franz Scherr[28], Wolfgang Maass[28], Srikanth Ramaswamy[29], Jonathan Tapson[30], Priyadarshini Panda[31], Youngeun Kim[31], Gouhei Tanaka[32], Simon Thorpe[33], Chiara Bartolozzi[34], Thomas A Cleland[35], Christoph Posch[36], ShihChii Liu[18], Gabriella Panuccio[37], Mufti Mahmud[38], Arnab Neelim Mazumder[39], Morteza Hosseini[39], Tinoosh Mohsenin[39], Elisa Donati[18], Silvia Tolu[14], Roberto Galeazzi[40], Martin Ejsing Christensen[41], Sune Holm[42], Daniele Ielmini[43] and N Pryds[1]

Training Spiking Neural Networks Using Lessons From Deep Learning

Spike Trains Data/Spiking NN ⟷ Statistics

- **Neural Networks**

  attack problems with 10 million parameters and try to get an answer

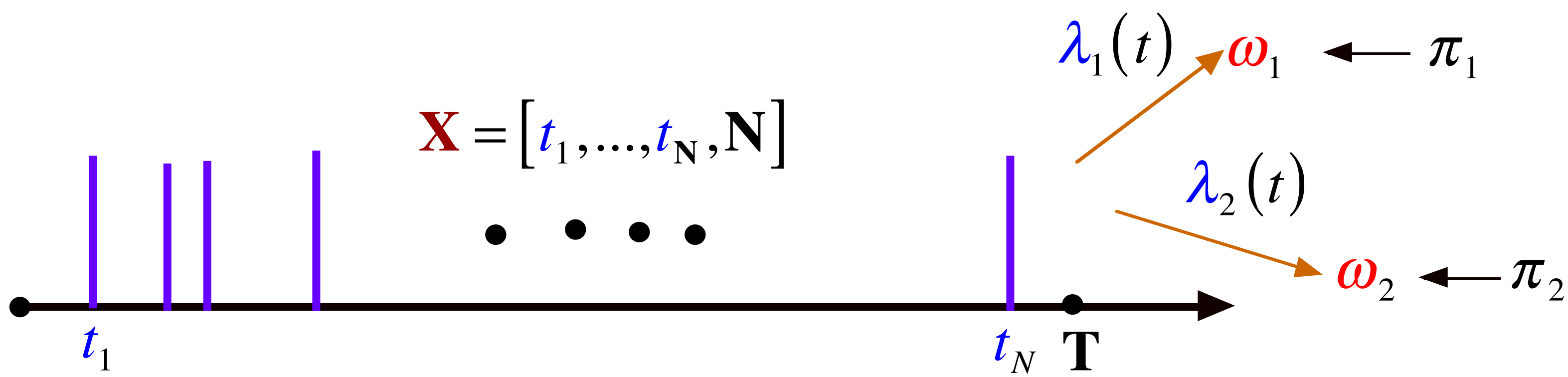
- **Statistics**

  attack problems with a simple model and try to get it right

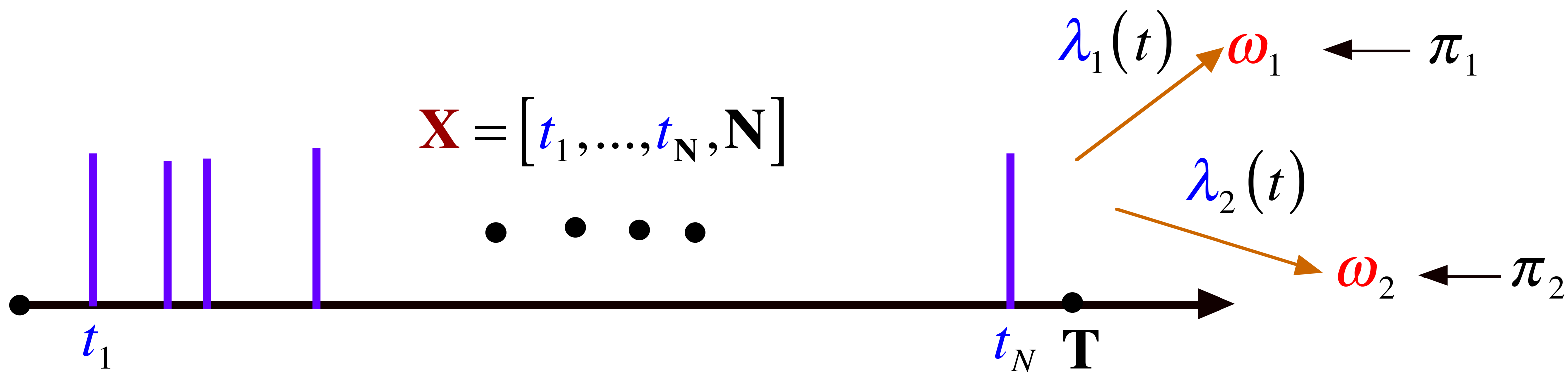
II Classification for Point Processes (Spike Trains Data)

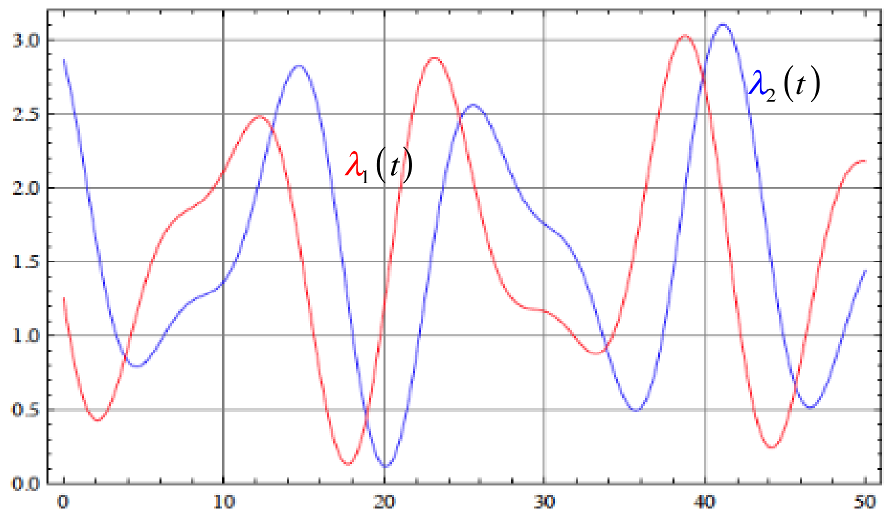
1. Bayes Rule and Risk

Classification Problem



Classification Problem

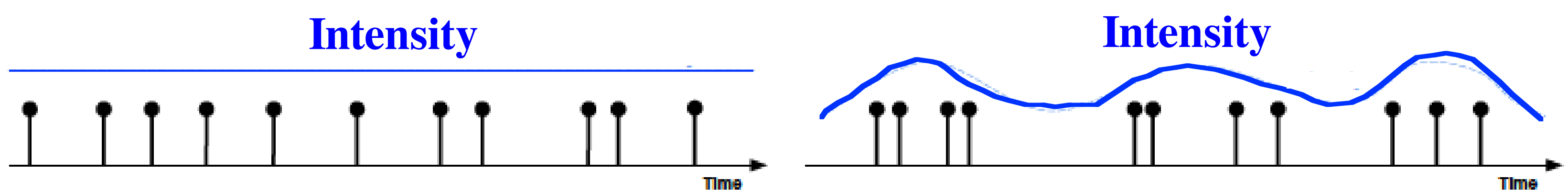




$$\mathbf{Pr}\left[\text{spike in } (t, t+\Delta t) \mid \mathbf{X} \in \omega_1\right] \approx \lambda_1(t)\Delta t$$

$$\mathbf{Pr}\left[\text{spike in } (t, t+\Delta t) \mid \mathbf{X} \in \omega_2\right] \approx \lambda_2(t)\Delta t$$

Intensity Function



$$\mathbf{E}\left[\mathbf{N}\,(t,t+\Delta t)\mid \mathbf{H}(t)\right]\approx \lambda\left(t\mid \mathbf{H}(t)\right)\Delta t$$

$\mathbf{H}(t)$ - the history of all events up to time t

The Likelihood Function

The occurrence density of counting process $\mathbf{N}(t)$ with the intensity function $\lambda(t)$

$$\mathbf{X} = [t_1, \ldots, t_{\mathbf{N}}, \mathbf{N}]$$

$$\boxed{\mathbf{N} = \mathbf{N}(T)}$$

$$\boxed{f(\mathbf{x}) = \prod_{i=1}^{\mathbf{N}} \lambda(t_i) \exp\left(-\int_0^T \lambda(u)\,du\right)}$$

***Daley, Vere-Jones: An Introduction to the Theory of Point Processes**

Bayes Rule

▲ **Bayes Rule in Terms of Local Intensities**

$\boxed{\pi_1 = \pi_2}$

$$\psi_T^* : \mathbf{X} \in \omega_1 \quad \textbf{if} \quad \sum_{i=1}^{\mathbf{N}} \log\left(\frac{\lambda_1(t_i)}{\lambda_2(t_i)}\right) \geq \gamma$$

$$\uparrow$$

$$\gamma = \int_0^T \left(\lambda_1(u) - \lambda_2(u)\right) du$$

• $\gamma = \int_0^T \left(\lambda_1(u) - \lambda_2(u)\right) du + \log\left(\frac{\pi_2}{\pi_1}\right)$ •

▲ Bayes Rule in Terms of Stochastic Integrals

$$\psi_T^* : \mathbf{X} \in \omega_1 \quad \text{if} \quad \int_0^T \log\left(\frac{\lambda_1(t)}{\lambda_2(t)}\right) d\mathbf{N}(t) \geq \gamma$$

A1:

- $\lambda_i(t)$ **- deterministic functions,** $i = 1,2$
- $d\mathbf{N}_i(t) = \lambda_i(t)dt + d\mathbf{M}_i(t)$
- $\mathbf{M}_i(t) = \mathbf{N}_i(t) - \int_0^t \lambda_i(s)ds$ **- martingale**

$$\Downarrow$$

- **Poisson processes with intensity functions** $\lambda_i(t)$, $i = 1,2$. (**Watanabe Th.**)

▲ Bayes Rule in Terms of Stochastic Integrals

$$\psi_T^* \ : \ \mathbf{X} \in \omega_1 \quad \textbf{if} \quad U_T(\mathbf{X}) \geq \alpha_T$$

- $U_T(\mathbf{X}) = \int_0^T \log\left(\frac{\lambda_1(t)}{\lambda_2(t)}\right) d\mathbf{M}(t)$

  zero mean martingale

- $\alpha_T = \int_0^T \left(\lambda_1(t) - \lambda_2(t)\right) dt + \int_0^T \log\left(\frac{\lambda_1(t)}{\lambda_2(t)}\right) \lambda(t)\, dt$

$$\lambda(t) = \begin{cases} \lambda_1(t) & \text{if } \mathbf{X} \in \omega_1 \\ \lambda_2(t) & \text{if } \mathbf{X} \in \omega_2 \end{cases}$$

▲ Shape Densities & Intensity Factors

- $$\lambda_1(t) = \underbrace{\boxed{\frac{\lambda_1(t)}{\int_0^T \lambda_1(u)\,du}}}_{p_1(t)} \underbrace{\boxed{\int_0^T \lambda_1(u)\,du}}_{\tau_1}$$ $$\boxed{\tau_1 = \mathbf{E}\left[\mathbf{N}_1(T)\right]}$$

- $$\lambda_2(t) = p_2(t)\tau_2$$ $$\boxed{\tau_2 = \mathbf{E}\left[\mathbf{N}_2(T)\right]}$$

Example: $\lambda_1(t) = \lambda(t)$ **and** $\lambda_2(t) = \mu\lambda(t)$ $\boxed{\mu > 0}$

$$\Rightarrow p_1(t) = p_2(t), \quad \tau_2 = \mu\tau_1$$

▲ **Bounds** $U_T(\mathbf{X}) \geq \alpha_T$

$$\mathbf{K}_T(p \parallel q) = \int_0^T \log\left(\frac{p(t)}{q(t)}\right) p(t)\,dt$$

$\mathbf{K}_T(\bullet \parallel \bullet)$: **KL divergence**

- $\mathbf{X} \in \omega_1:\ -\dfrac{(\tau_1 - \tau_2)^2}{\tau_2} - \tau_1 \mathbf{K}_T(p_1 \parallel p_2) \leq \alpha_T \leq -\tau_1 \mathbf{K}_T(p_1 \parallel p_2)$

- $\mathbf{X} \in \omega_2:\ \tau_2 \mathbf{K}_T(p_2 \parallel p_1) \leq \alpha_T \leq \dfrac{(\tau_1 - \tau_2)^2}{\tau_1} + \tau_2 \mathbf{K}_T(p_2 \parallel p_1)$

Note: $\tau_1 = \tau_2 = \tau \Rightarrow$

$$\omega_1:\ \alpha_T = -\tau \mathbf{K}_T(p_1 \parallel p_2)$$

$$\omega_2:\ \alpha_T = \tau \mathbf{K}_T(p_2 \parallel p_1)$$

▲ **Bounds** $U_T(\mathbf{X}) \geq \alpha_T$

$$U_T(\mathbf{X}) = \int_0^T \log\left(\frac{\lambda_1(t)}{\lambda_2(t)}\right) dM(t)$$

$\mathbf{V}_T(p \,\|\, q) = \int_0^T \log^2\left(\frac{p(t)}{q(t)}\right) p(t)\,dt$: **KL variation**

$\mathbf{K}_T(p \,\|\, q) \leq \sqrt{\mathbf{V}_T(p \,\|\, q)}$

- $\mathbf{X} \in \omega_1$:

$$\tau_1 \left\{ \mathbf{K}_T(p_1 \,\|\, p_2) + \log\left(\frac{\tau_1}{\tau_2}\right) \right\}^2 \leq \mathrm{var}\left[U_T(\mathbf{X})\right] \leq \tau_1 \left\{ \sqrt{\mathbf{V}_T(p_1 \,\|\, p_2)} + \log\left(\frac{\tau_1}{\tau_2}\right) \right\}^2$$

- $\mathbf{X} \in \omega_2$:

$$\tau_2 \left\{ \mathbf{K}_T(p_2 \,\|\, p_1) + \log\left(\frac{\tau_2}{\tau_1}\right) \right\}^2 \leq \mathrm{var}\left[U_T(\mathbf{X})\right] \leq \tau_2 \left\{ \sqrt{\mathbf{V}_T(p_2 \,\|\, p_1)} + \log\left(\frac{\tau_2}{\tau_1}\right) \right\}^2$$

Note:

$$\textbf{A2}: \ 0 < \delta \le \lambda_i(t) \le C, \text{ for all } t \ge 0$$

- $\mathbf{X} \in \omega_1$: $\tau_1 \log^2\left(\frac{\delta}{C}\right) \le \text{var}\left[U_T(\mathbf{X})\right] \le \tau_1 \log^2\left(\frac{C}{\delta}\right)$
- $\mathbf{X} \in \omega_2$: $\tau_2 \log^2\left(\frac{\delta}{C}\right) \le \text{var}\left[U_T(\mathbf{X})\right] \le \tau_2 \log^2\left(\frac{C}{\delta}\right)$

▲ **Asymptotic**

$$\textbf{A3}: \frac{1}{T}\int_0^T \lambda_i(s)\,ds \to d > 0 \text{ as } T \to \infty,\ i = 1,2$$

$$d\log^2\left(\frac{\delta}{C}\right) \le \underline{\lim}_T \operatorname{var}\left[\frac{1}{\sqrt{T}}U_T(\mathbf{X})\right] \le \overline{\lim}_T \operatorname{var}\left[\frac{1}{\sqrt{T}}U_T(\mathbf{X})\right] \le d\log^2\left(\frac{C}{\delta}\right)$$

Bayes Risk

$$\mathbf{R}_T^* = \frac{1}{2}\mathbf{P}\left[U_T(\mathbf{X}) \geq \alpha_T \mid \mathbf{X} \in \omega_2\right] + \frac{1}{2}\mathbf{P}\left[U_T(\mathbf{X}) < \alpha_T \mid \mathbf{X} \in \omega_1\right]$$

Lemma (Le Guevel, 2021, CRM): Let $\mathbf{N}(t)$ be the counting process allowing the **martingale** decomposition

$$d\mathbf{N}(t) = \lambda(t)dt + d\mathbf{M}(t)$$

$$\mathbf{P}\left[\left|\int_0^T g(t)dM(t)\right| \geq \varepsilon\right] \leq 2\exp\left[-\frac{\varepsilon^2}{2\mathrm{v}_T + \varepsilon \mathrm{u}_T}\right]$$

$$\int_0^T g^2(t)\lambda(t)dt \leq \mathrm{v}_T \qquad \left|g(t)\right| \leq u_T$$

Theorem 1

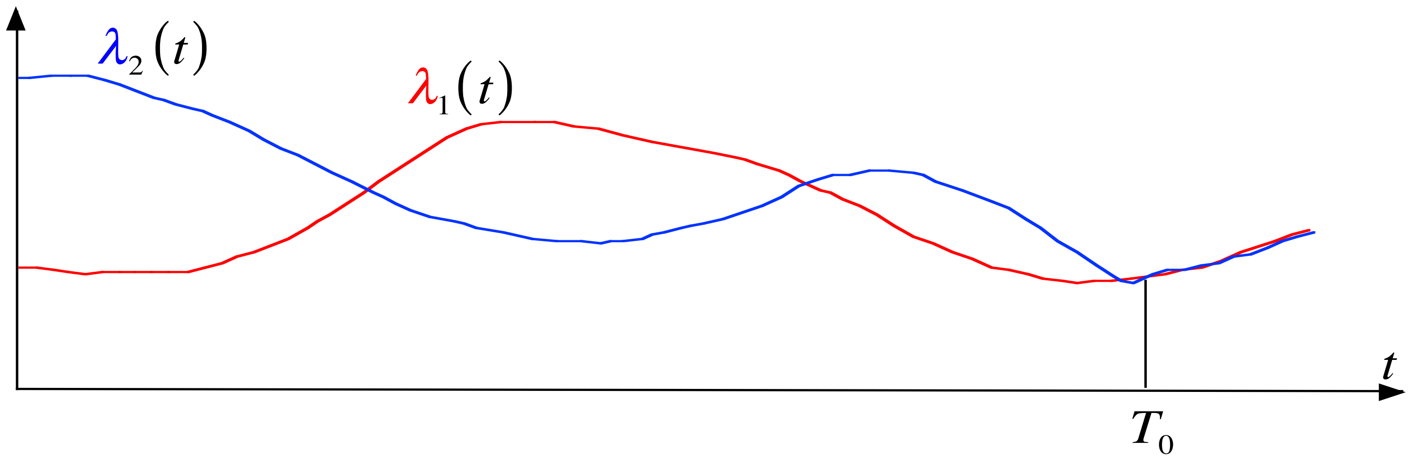
- **Under A1 + A2+ A3 + A4**

$$\boxed{\mathbf{R}_T^* \prec \frac{1}{2}\exp\left(-C_T T\right)}$$

- $C_T \approx \dfrac{1}{3} d$

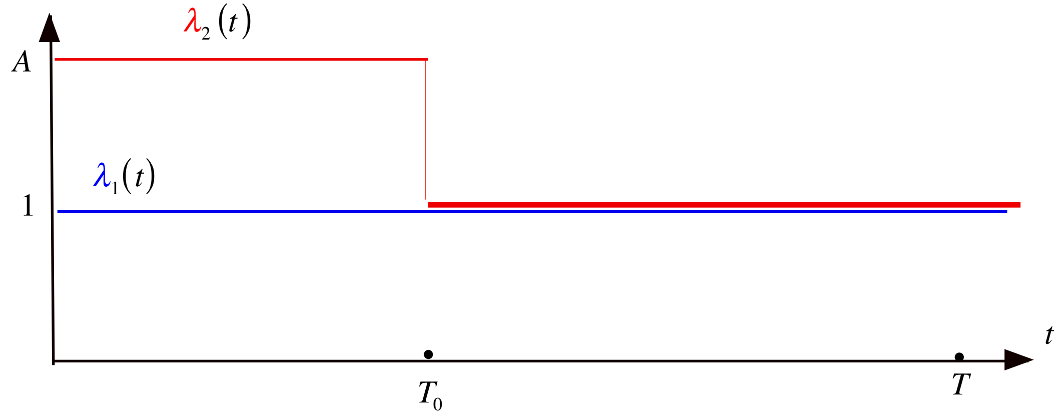
Assumption A4

$$\int_{T_0}^{\infty} \left| \lambda_1(s) - \lambda_2(s) \right| ds > 0$$



$$\int_{T_0}^{\infty} \left| \lambda_1(s) - \lambda_2(s) \right| ds = 0$$

Example



$$\mathbf{X} \in \omega_2 \quad \textbf{if} \quad \prod_{i=1}^{N(T)} \frac{\lambda_2(t_i)}{\lambda_1(t_i)} \exp\left[\int_0^T \lambda_1(s)\,ds - \int_0^T \lambda_2(s)\,ds\right] \geq 1$$

$$\alpha(T) = \mathbf{P}\left[\prod_{i=1}^{N(T)} \frac{\lambda_2(t_i)}{\lambda_1(t_i)} \exp\left[\int_0^T \lambda_1(s)\,ds - \int_0^T \lambda_2(s)\,ds\right] \geq 1 \mid \mathbf{X} \in \omega_1\right]$$

$$\alpha(T) = \mathbf{P}\left[N(T_0) > \frac{(A-1)T_0}{\log(A)} \mid \mathbf{X} \in \omega_1 \right]$$

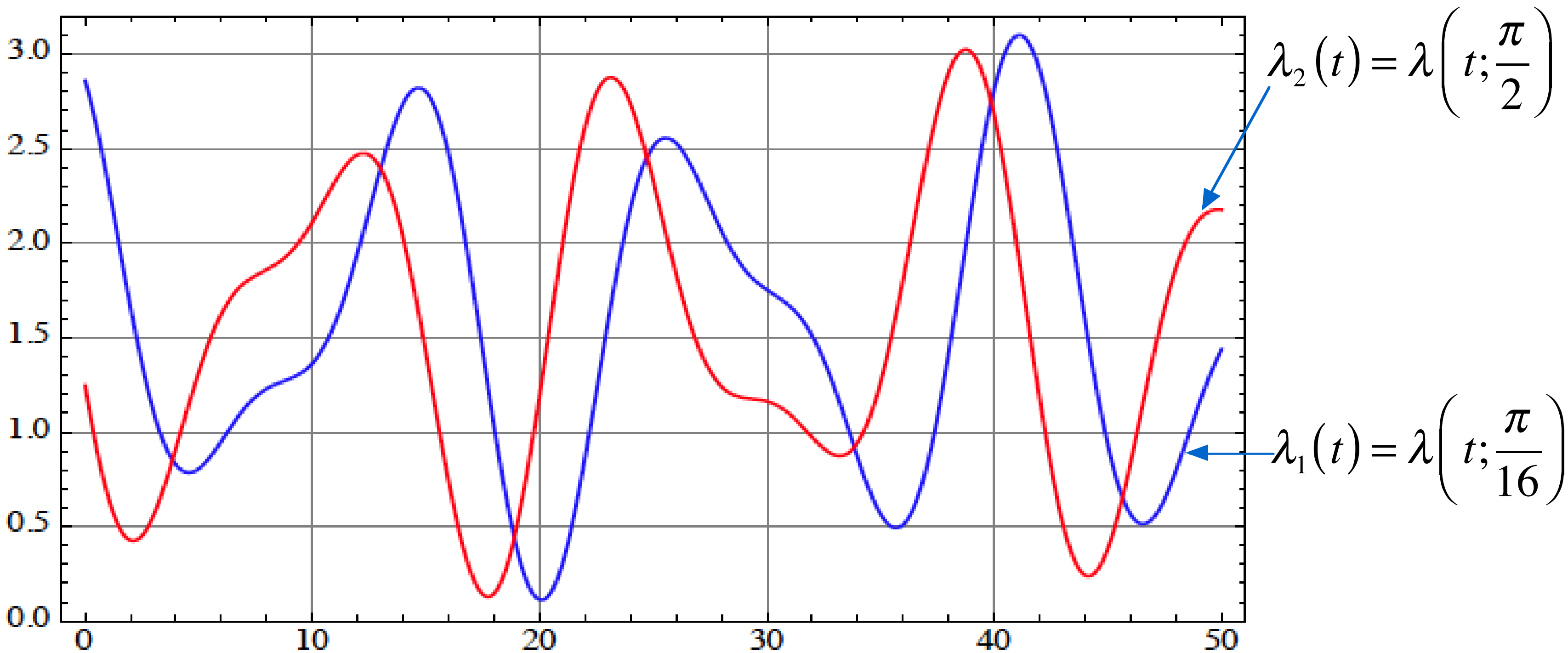
$$\Downarrow$$

$$\alpha(T) = \sum_{j > \frac{(A-1)T_0}{\log(A)}} \frac{1}{j!} T_0^j e^{-T_0}$$

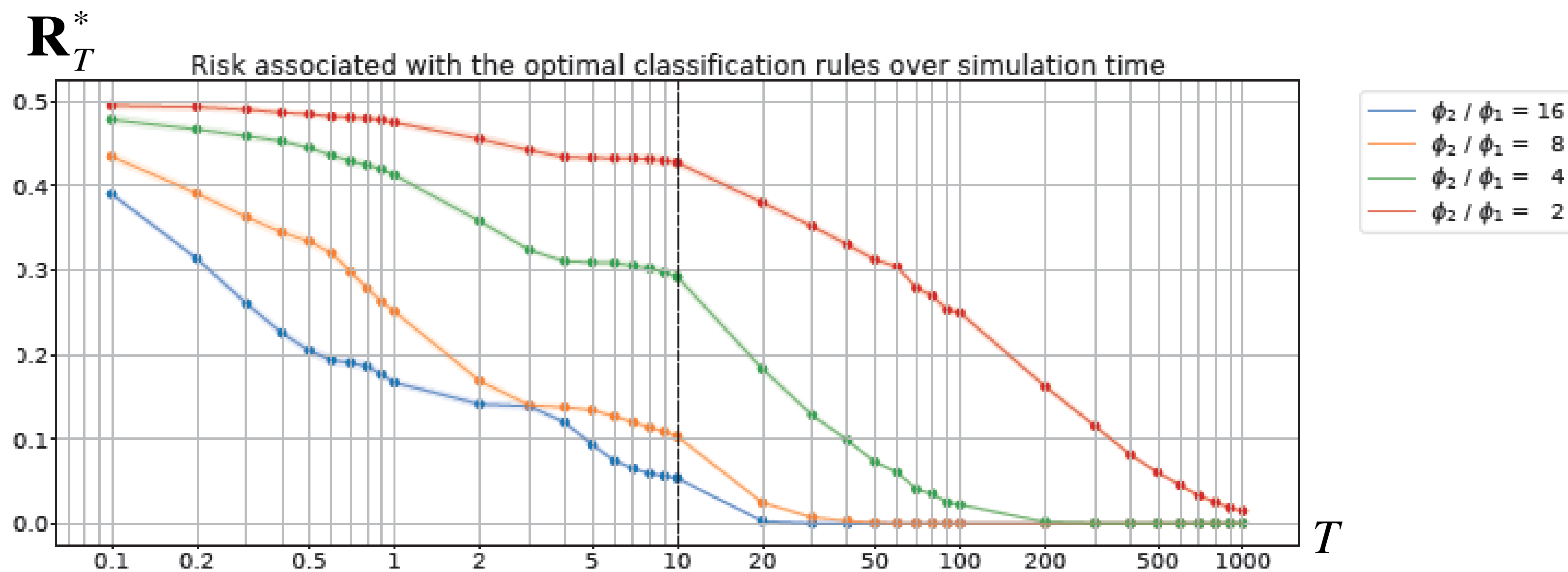
$\alpha(T)$ **is a constant for** $T > T_0$

Example: • $\lambda(t;\phi) = 1.6 + \cos\left(\frac{\pi}{4\sqrt{3}}t + \phi\right) + 0.5\cos\left(\frac{\pi}{3\sqrt{2}}t + \frac{\pi}{4} + \phi\right)$

$$\boxed{\lambda_1(t) = \lambda(t;\phi_1),\ \lambda_2(t) = \lambda(t;\phi_2)}$$



$$\lambda_1(t) = \lambda(t;\phi_1),\ \lambda_2(t) = \lambda(t;\phi_2)$$



- The slow decay of $\mathbf{R}_T^*$ for **close** intensities: $\phi_2 / \phi_1 = 2$.

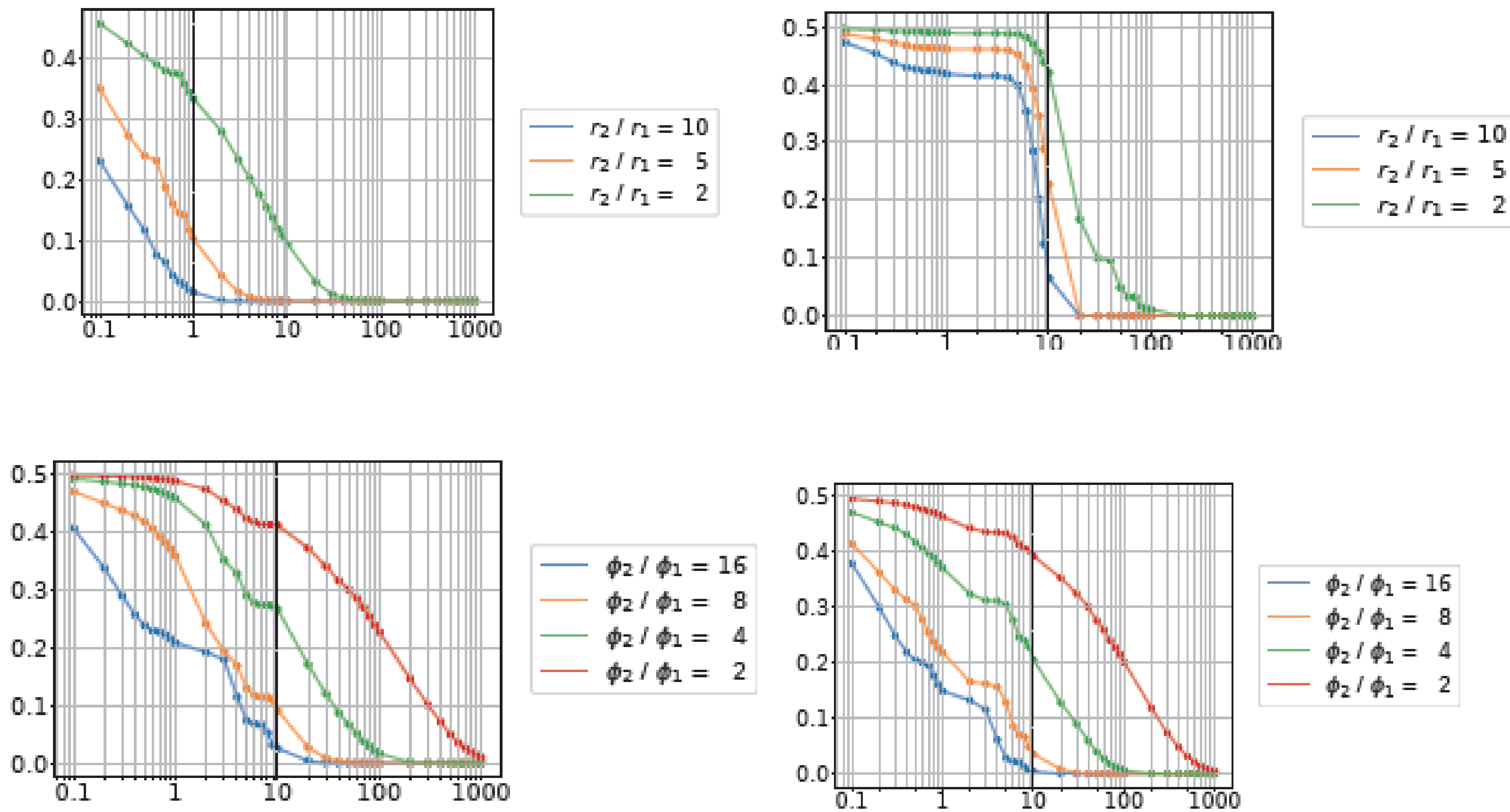
- The fast rate for **distant** intensities: $\phi_2 / \phi_1 = 16$.

- $$\lambda_1(t) = \lambda(t;\phi_1),\ \lambda_2(t) = \lambda(t;\phi_2)$$

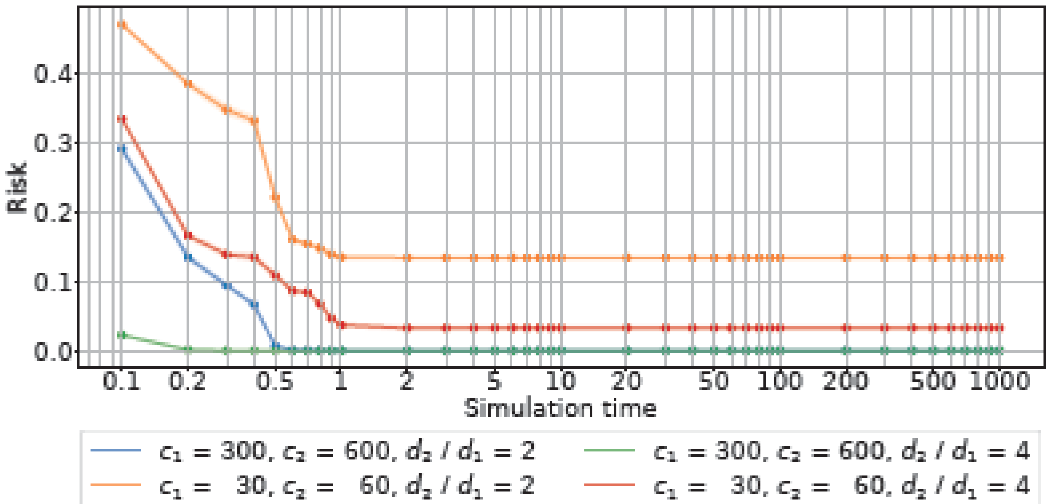
$$\lambda_1(t) = \lambda(t;r_1),\ \lambda_2(t) = \lambda(t;r_2)$$

a)	$\lambda(t;r) = r$
b)	$\lambda(t;r) = (r+1) + r\cos\left(\frac{2\pi}{30}t + 2.6\right) + \cos\left(\frac{2\pi}{28}t + 4.5\right)$
c)	$\lambda(t;\phi) = \left[3.1 + 3\cos\left(\frac{\pi}{3\sqrt{2}}t + \phi\right)\right]^{1/2}$
d)	$\lambda(t;\phi) = 1.3\exp\left[\cos\left(\frac{\pi}{3\sqrt{2}}t + \phi\right)\right]$

$\mathbf{R}_T^*$ versus T



- $\lambda(t;c,d) = c\exp\left[-d(t-0.5)^2\right] \longleftarrow \int_0^\infty \lambda_i(t)dt < \infty$



- $\underline{\lim}_T \mathbf{R}_T^* > 0$

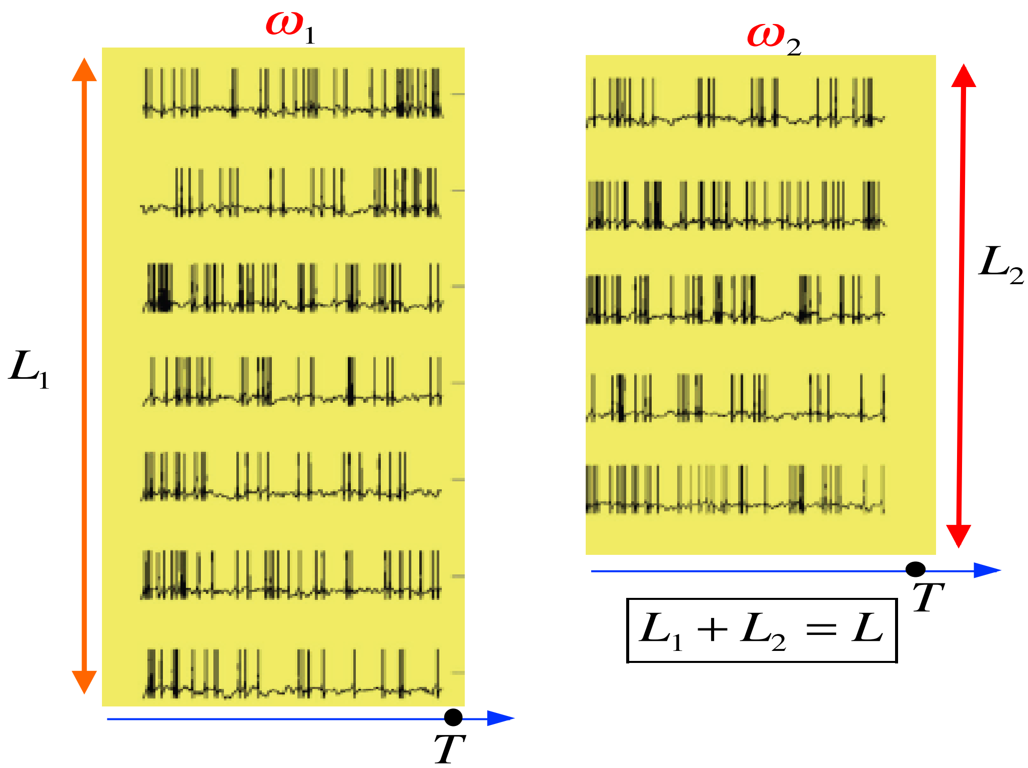
3. Nonparametric Classification Rules

▲ **Bayes Rule** $\psi_T^* : \mathbf{X} \in \omega_1$ **if** $\mathbf{W}_T(\mathbf{X}) \geq \eta_T$

- $\eta_T = \tau_1 - \tau_2 + N \log\left(\dfrac{\tau_2}{\tau_1}\right)$

- $\mathbf{W}_T(\mathbf{X}) = \sum_{i=1}^{N} \log\left(\dfrac{p_1(t_i)}{p_2(t_i)}\right)$

▲ **Training Set:** $\mathbf{D}_L = \{(\mathbf{X}_1, Y_1), \ldots, (\mathbf{X}_L, Y_L)\}$



$$\mathbf{X}_j = \left[t_1^{[j]}, \ldots, t_{\mathbf{N}^{[j]}}^{[j]}, \mathbf{N}^{[j]} \right] \quad , \quad Y_j \in \{\omega_1, \omega_2\}$$

▲ Plug-In Rules

$$\widehat{\psi}_{L,T} : \mathbf{X} \in \omega_1 \ \ \textbf{if} \ \ \widehat{\mathbf{W}}_{L,T}(\mathbf{X}) \geq \hat{\eta}_{L,T}$$

$$\mathbf{X} = [t_1, \ldots, t_{\mathbf{N}}, \mathbf{N}]$$

- $\hat{\eta}_{L,T} = \hat{\tau}_1 - \hat{\tau}_2 + N \log\left(\frac{\hat{\tau}_2}{\hat{\tau}_1}\right)$

- $\widehat{\mathbf{W}}_{L,T}(\mathbf{X}) = \sum_{i=1}^{N} \log\left(\frac{\hat{p}_1(t_i)}{\hat{p}_2(t_i)}\right)$

▲Estimates Based on Data Aggregation

- $$\hat{\tau}_1 = \frac{1}{L_1}\sum_{j=1}^{L} N^{[j]}\mathbf{1}\left(Y_j = \omega_1\right)$$

  $$\hat{\tau}_2 = \frac{1}{L_2}\sum_{j=1}^{L} N^{[j]}\mathbf{1}\left(Y_j = \omega_2\right)$$

- $$\hat{p}_1(t) = \frac{1}{L_1}\sum_{j=1}^{L} \hat{p}_1^{[j]}(t)\mathbf{1}\left(Y_j = \omega_1\right)$$

  $$\hat{p}_2(t) = \frac{1}{L_2}\sum_{j=1}^{L} \hat{p}_2^{[j]}(t)\mathbf{1}\left(Y_j = \omega_2\right)$$

▲ Bayes Risk Consistency

- **The Conditional Risk**

$$\mathbf{R}_{L,T} = \mathbf{P}\left[\widehat{\psi}_{L,T}(\mathbf{X}) \neq Y \mid \mathbf{D}_L \right]$$

- **BRC**

$$\mathbf{R}_{L,T} \rightarrow \mathbf{R}_T^* \ \ \textbf{as} \ \ L \rightarrow \infty \ \ (P)$$

Theorem 2

Consider the plug-in classification rules such that

$$\sup_{t\in[0,T]}\left|\hat{p}_i(t)-p_i(t)\right|\to 0 \textbf{ as } L\to\infty \quad (P) \quad i=1,2$$

Then

$$\mathbf{R}_{L,T}\to\mathbf{R}_T^* \textbf{ as } L\to\infty \ (P)$$

Note: Helly's Theorem for the Stieltjes Integral

$$f_n(x)\to f(x) \textbf{ uniformly on } [0,T] \textbf{ and } g\in\mathbf{BV}[0,T]$$

$$\Rightarrow \int_0^T f_n(x)dg \to \int_0^T f(x)dg$$

4. Kernel Classifiers

- **The kernel estimate from the single realization**

$$\hat{p}_i^{[j]}(t) = \frac{1}{N^{[j]}} \sum_{l=1}^{N^{[j]}} K_h\left(t - t_l^{[j]}\right)$$

$$\boxed{K_h(t) = \frac{1}{h} K\left(\frac{t}{h}\right)}$$

- **The kernel estimate from the aggregated data**

$$\hat{p}_i(t) = \frac{1}{L_i} \sum_{j=1}^{L} \hat{p}_i^{[j]}(t) \mathbf{1}\left(Y_j = \omega_i\right), \quad i = 1,2$$

- **Uniform Convergence for the Kernel Estimate**

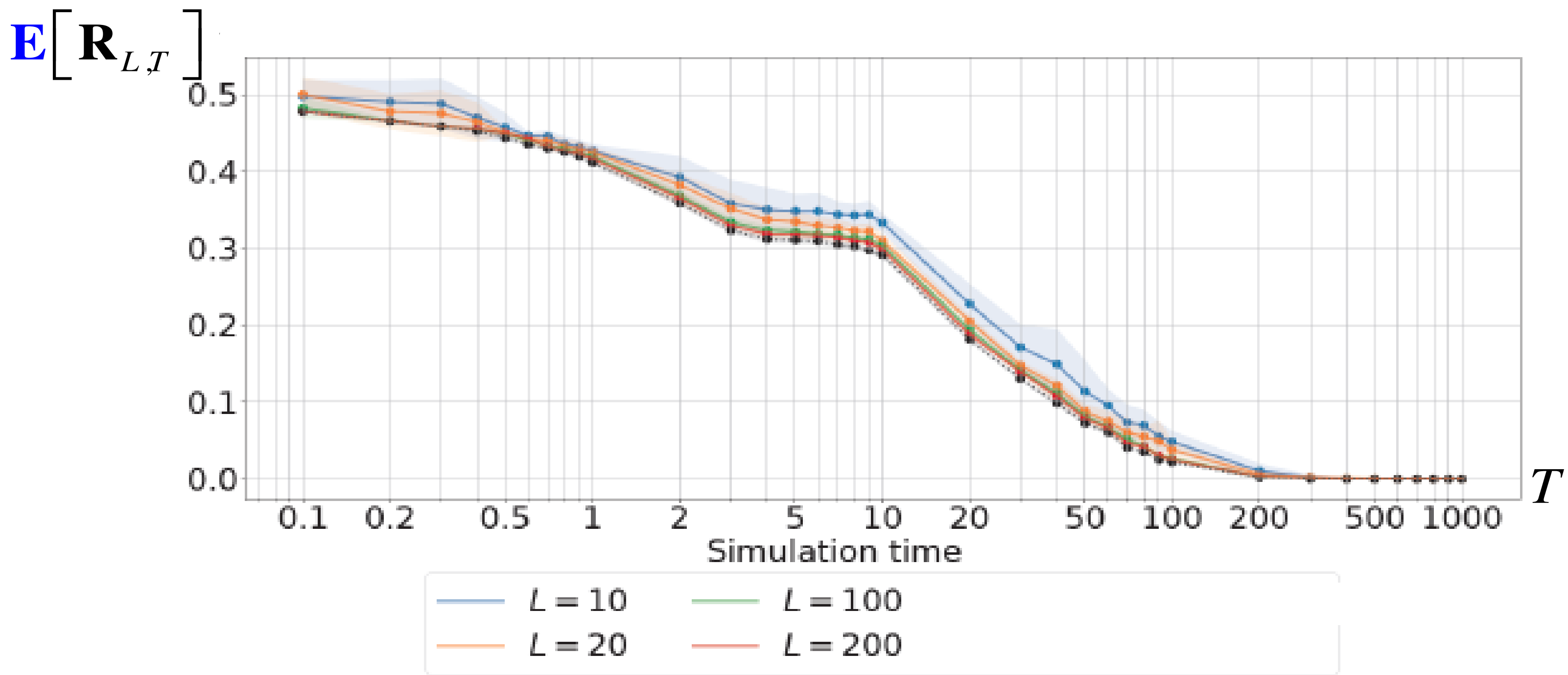
  - $\lambda_i(t)$ **- Lipschitz on** $\mathbb{R}_+$
  - $K(t)$ **- Lipschitz on** $[-1,1]$
  - $h(L) \to 0$ **and** $\dfrac{Lh(L)}{\log(L)} \to \infty$

Then

$$\sup_{t\in[\varepsilon,T-\varepsilon]}\left|\widehat{p}_i(t)-p_i(t)\right| \to 0 \text{ as } L\to\infty \quad (P) \quad i=1,2$$

Note: Need the boundary corrected kernels

Example: $\lambda_1(t)=\lambda(t;\phi_1)$, $\lambda_2(t)=\lambda(t;\phi_2)$, $\phi_2/\phi_1=4$.



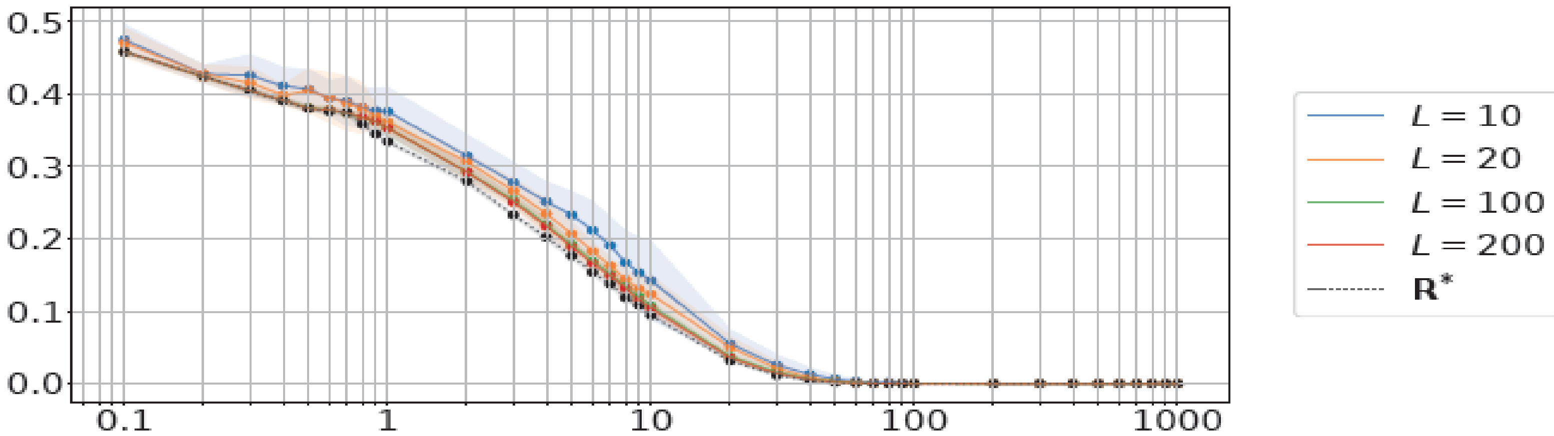
- h → **data driven choice by max log-likelihood**

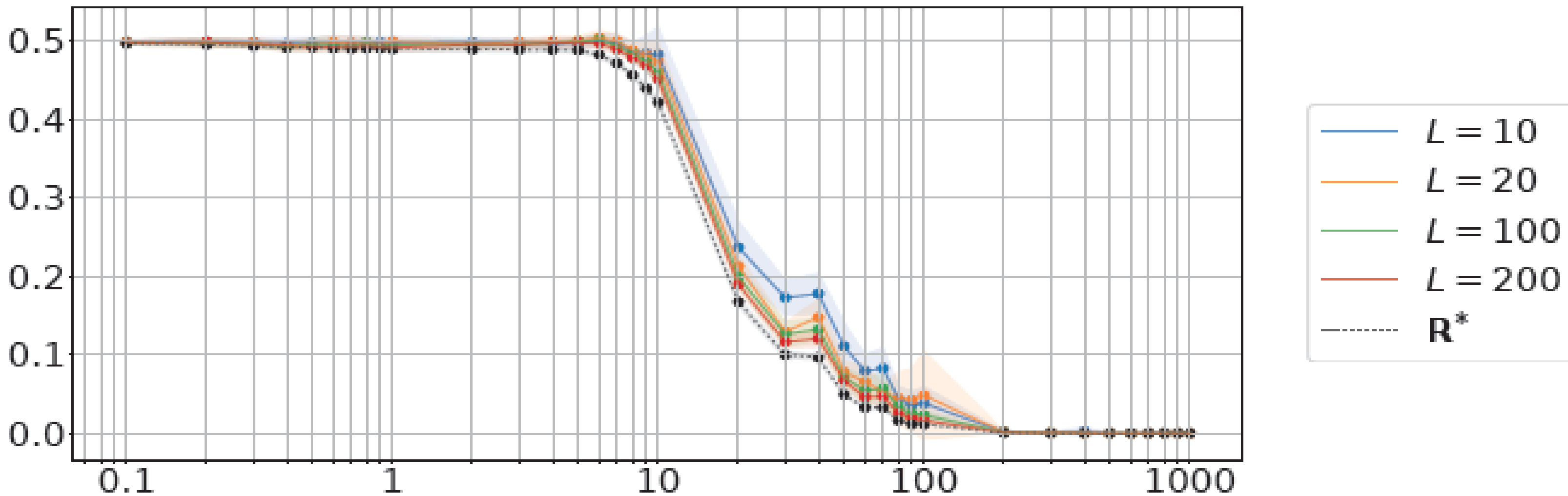
- $$\lambda_1(t) = \lambda(t;\phi_1),\ \lambda_2(t) = \lambda(t;\phi_2)$$

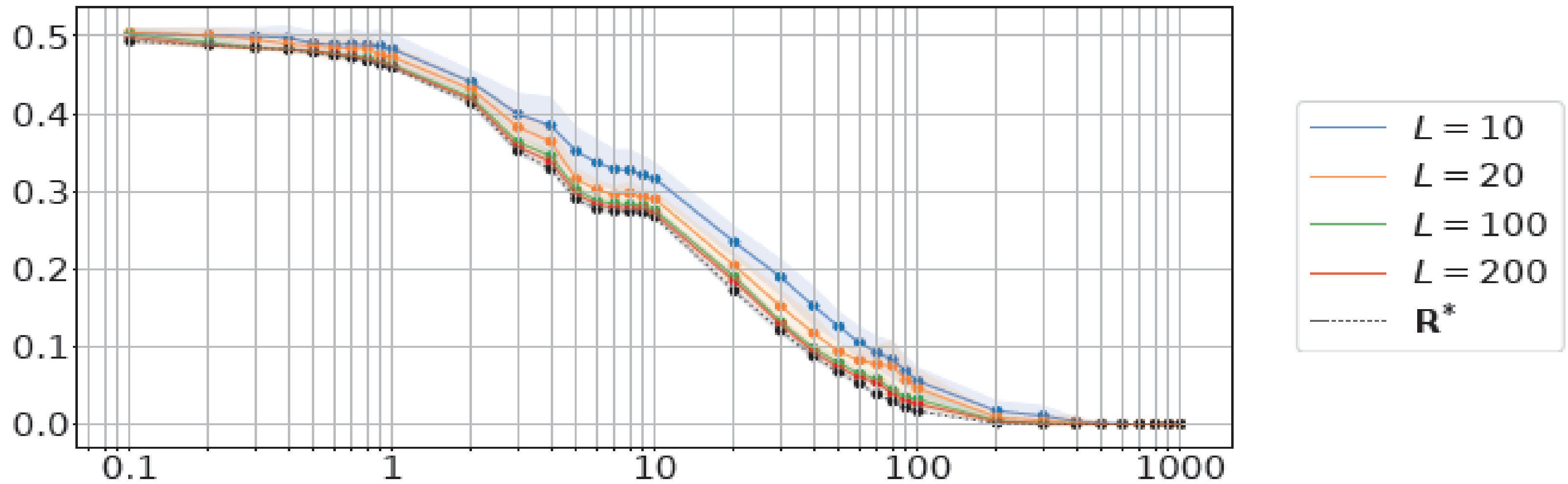
$$\lambda_1(t) = \lambda(t;r_1),\ \lambda_2(t) = \lambda(t;r_2)$$

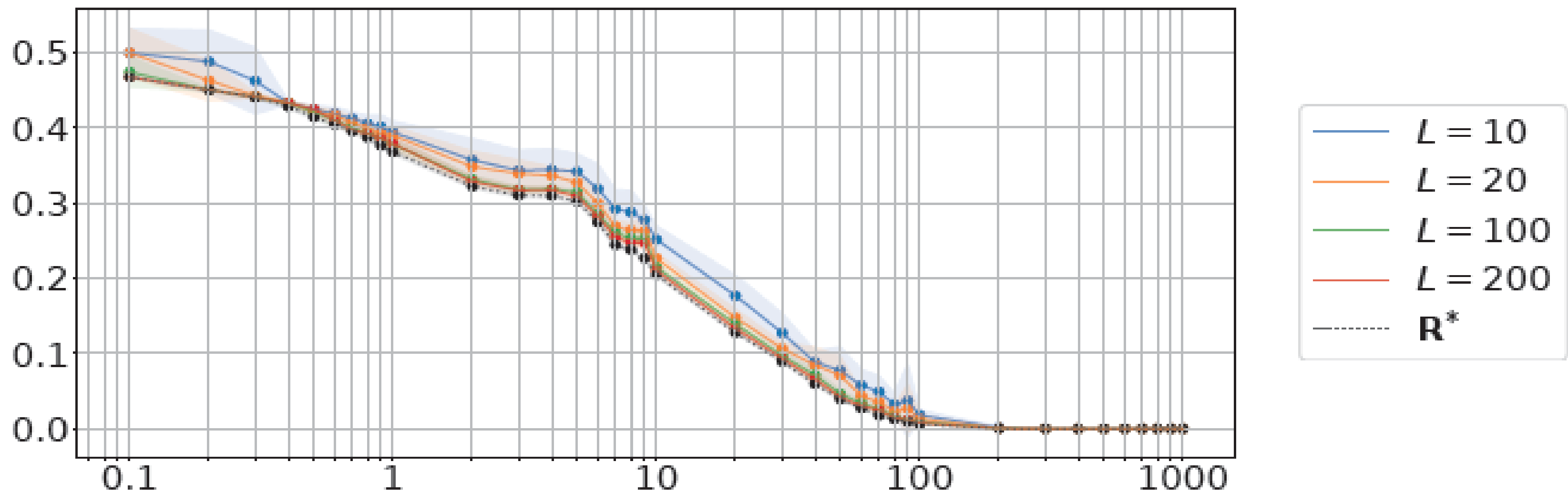
a)	$\lambda(t;r) = r$
b)	$\lambda(t;r) = (r+1) + r\cos\left(\frac{2\pi}{30}t + 2.6\right) + \cos\left(\frac{2\pi}{28}t + 4.5\right)$
c)	$\lambda(t;\phi) = \left[3.1 + 3\cos\left(\frac{\pi}{3\sqrt{2}}t + \phi\right)\right]^{1/2}$
d)	$\lambda(t;\phi) = 1.3\exp\left[\cos\left(\frac{\pi}{3\sqrt{2}}t + \phi\right)\right]$

$\mathbf{E}\left[\mathbf{R}_{L,T}\right]$ versus T






0.5
0.4
0.3
0.2
0.1
0.0
0.1
1
10
100
1000
$L = 10$
$L = 20$
$L = 100$
$L = 200$
$\mathbf{R}^*$


0.5
0.4
0.3
0.2
0.1
0.0
0.1
1
10
100
1000
$L = 10$
$L = 20$
$L = 100$
$L = 200$
$\mathbf{R}^*$

5. Extensions

▲ **Rates !**

$$\mathbf{R}_{L,T} = \mathbf{R}_T^* + O_P\left(L^{-\alpha}\right)$$

▲ **Inference from the Single Realization**

- **Aalen Multiplicative Models**

O. Aalen. Nonparametric inference for a family of counting processes. The Annals of Statistics, 1978.

$$\lambda_i(t) = \mu\gamma_i(t), \ \mu \nearrow$$

▲ **Multi-Class Problem**: $\left(\omega_1, \lambda_1(t)\right), \ldots, \left(\omega_c, \lambda_c(t)\right)$

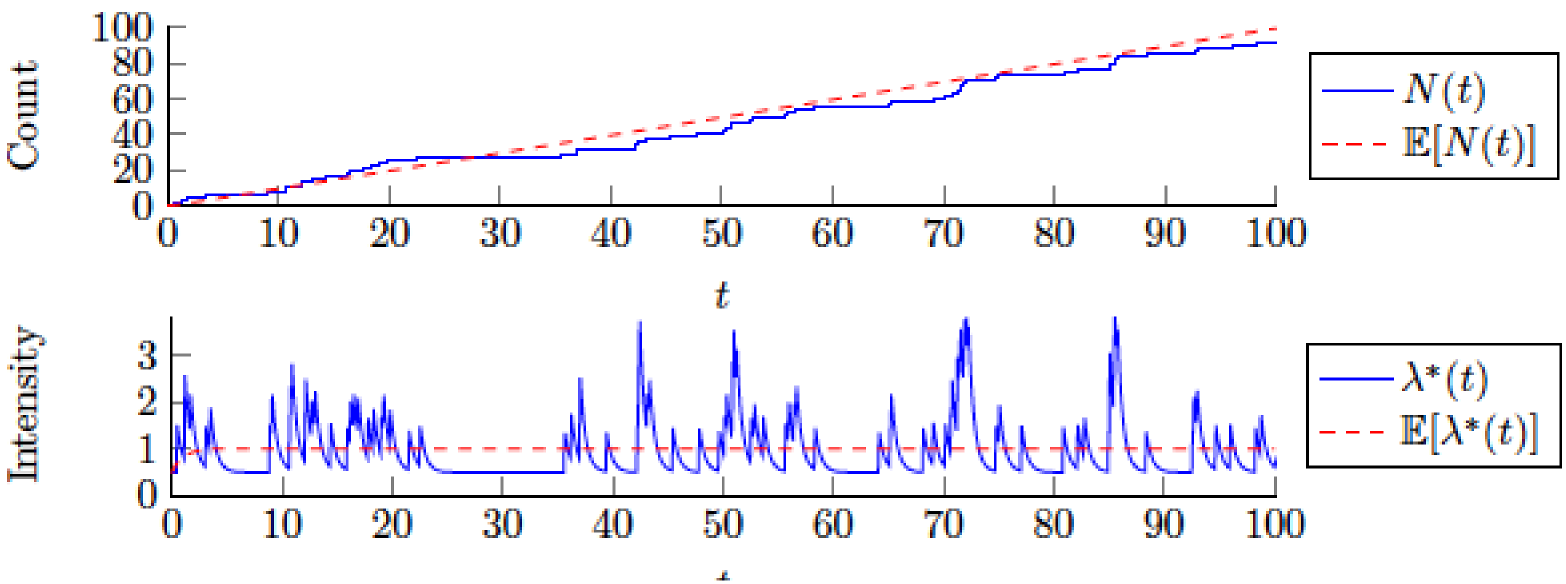
$$\psi_T^* : \mathbf{X} \in \omega_i \quad \textbf{if} \quad \sum_{s=1}^{\mathbf{N}} \log\left(\frac{\lambda_i(t_s)}{\lambda_k(t_s)}\right) \geq \gamma_{ik} \quad \textbf{for all} \quad k \neq i$$

$$\uparrow$$

$$\gamma_{ik} = \int_0^T \left(\lambda_i(u) - \lambda_k(u)\right) du + \log\left(\frac{\pi_k}{\pi_i}\right)$$

▲ Random Intensities

$$\lambda^*(t) = \lambda_0 + \sum_{t_i < t} g(t - t_i) = \lambda_0 + \int_0^t g(t-s) d\mathbf{N}(s)$$



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Dziękuję !